B.Sc./2nd Sem (H)/MTM/25(CBCS)

**2025**

# 2nd Semester Examination

## MATHEMATICS (Honours)

**Paper : GE 2-T**

**[Algebra]**

[CBCS]

Full Marks : 60 Time : Three Hours

*The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.*

1. Answer any *ten* questions : 2×10=20

(a) Prove that $1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots+\frac{x^n}{n!}=0$ cannot have multiple root.

(b) If $x$, $y$ are non-zero complex numbers, prove that $\log\ xy \neq \log x+\log y$.

(c) Prove that for all $n\in\mathbb{N}$, $(3+\sqrt{5})^n+(3-\sqrt{5})^n$ is an even integer.

(d) Let $f:\mathbb{R}\to\mathbb{R}$ be a mapping such that $f(x)=\frac{1}{2}(e^x+e^{-x})$, $x\in R$. Find a restriction of $f$ which is bijective.

P.T.O.

(e) Find the condition that the roots of the eq
$x^3 - px^2 + qx - r = 0$ are in G.P.

(f) Let $A$ be a set of $n$ elements and $B$ be a
$m$ elements. Find the numbers of mapping fr
to $B$.

(g) If $n$ is a positive integer, show that $(n+1)^n > 2$

(h) Show that square of any integer is one of the fo
$3k$, $3k+1$.

(i) Apply Descartes' rule of sign to find the nature o
the roots $x^7 + x^5 - x + 3 = 0$.

(j) Find two integers $u$ and $v$ satisfying
$54u + 24v = 30$.

(k) Find the remainder when $3^{12} + 5^{12}$ is divided by
13.

(l) Show that there does not exist a linear map
$T : \mathbb{R}^5 \to \mathbb{R}^2$ whose kernel is
$\{(x_1, x_2, x_3, x_4, x_5) : x_1 = 3x_2 \text{ and } x_3 = x_4 = x_5\}$.

(m) Show that the map $T : \mathbb{R}^3 \to \mathbb{R}$ defined by
$T(x, y, z) = x + y + z$, for all $(x, y, z) \in \mathbb{R}^3$ is linear.

(n) Check whether the vectors (1,1,1), (1,1,0) and
(1,0,0) are linearly independent or not.



(o) Let $f : A \to B$ be an onto mapping and $S$, $T$ be subsets of $B$. Prove that

$$f^{-1}(S \cup T) = f^{-1}(S) \cup f^{-1}(T).$$

2. Answer any ***four*** questions : 5×4=20

(a) If $\tan(\theta + i\phi) = \sin(\alpha + i\beta)$, then show that

$$\coth\beta \sinh 2\phi = \cot\alpha \sin 2\theta.$$

(b) Solve the equation by Cardan's method :

$$x^3 - 15x^2 - 33x + 847 = 0.$$

(c) Find the eigenvalues and corresponding eigen vectors of the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(x, y, z) = (-x + 2y, 3y, 0)$.

(d) If $a$, $b$, $c$ are positive real numbers such that $a + b + c = 1$ then show that

$$\sqrt{4a+1} + \sqrt{4b+1} + \sqrt{4c+1} < 5.$$

(e) Suppose $f : A \to B$ and $g : B \to C$ be two maps. Define composition map of $f$ and $g$. Prove that if $g \circ f$ is bijective then $f$ is injective and $g$ is surjective. 1+(2+2)

(f) Find the relations among $q$, $p$, $r$ and $s$ that the product of two roots of the equations $x^4 + px^3 + qx^2 + rx + s = 0$ is unity.

P.T.O.

3. Answer any *two* questions : 10×2

(a) (i) $\begin{bmatrix}1\\2\\1\end{bmatrix}, \begin{bmatrix}0\\1\\1\end{bmatrix}, \begin{bmatrix}1\\1\\1\end{bmatrix}$ are the eigenvectors corresponding the eigenvalues 1,2,0 of a square matrix $A$ of order 3. Find $A$.

(ii) If $\alpha,\beta,\gamma$ be the roots of the equation $x^3+px^2+qx+r=0$, then form the equation whose roots are

$$\alpha+\frac{1}{\alpha}, \beta+\frac{1}{\beta}, \gamma+\frac{1}{\gamma}$$ 6+4

(b) (i) Find the roots of the equation $z^n=(z+1)^n$, where $z$ is a complex number and $n$ is a positive integer.

(ii) Verify whether the set $\{3x-1, x^3+1, x-3\}$ is a basis for the vector space $P_3(\mathbb{R})$.

(iii) Prove that $n^4+4^n$ is a composite number for all $n>1$.

3+3+4

(c) (i) For what values of $k$ the planes $x-4y+5z=k$, $x-y+2z=3$ and $2x+y+z=0$ intersect in a line. Find the equation of the intersecting line in that case.



(ii) The matrix of a linear mapping $T:\mathbb{R}^3\to\mathbb{R}^3$ relative to the ordered basis $\{(-1,1,1), (1,-1,1), (1,1,-1)\}$ of $\mathbb{R}^3$ is $\begin{bmatrix}1&2&2\\2&1&3\\3&3&1\end{bmatrix}$. Find the matrix of $T$ relative to the ordered basis $\{(0,1,1), (1,0,1), (1,1,0)\}$ of $\mathbb{R}^3$.

(4+1)+5

(d) (i) Find a basis and dimension of the subspace $S$ of $\mathbb{R}^4$ defined by

$$S=\left\{(x,y,z,w)\in\mathbb{R}^4 \,\middle|\, x+2y=z,\ 2x+y+w=0\right\}.$$

(ii) Let $A=\begin{pmatrix}7&-5&15\\6&-4&15\\0&0&1\end{pmatrix}$. Find a matrix $Q$ such that $Q^{-1}AQ$ is a diagonal matrix and hence calculate $A^6$. 5+5